

- 1) Find all prime ideals. and maximall ideals of  $Z_6, Z_2 \times Z_4$
- 2) Find all  $c \in Z_3$  such that  $Z_3[X] / \langle x^2 + c \rangle$  is a field .
- 3) show that  $N$  is a maximal ideal in a ring if and only if  $R/N$  is a simple ring .
- 4) let  $A$  and  $B$  be ideals of a commutative ring  $R$  . the quotient  $A : B$  of  $A$  by  $B$  is defined by  $A : B = \{r \in R : r b \in A \text{ for all } b \in B\}$   
show that  $A : B$  is an ideal of  $R$  .
- 5) Find all *zero* divisors ,and a nonzero , idempotent and units , nilpotent element in  $Z_3 \oplus Z_6$  .
- 6) suppose that  $a$  and  $b$  belong to a commutitve ring . and  $ab$  is a *zero* divisor. Show that either  $a$  orb is a *zero* divisor
- 7) Prove that  $I = \langle 2 + 2i \rangle$  is not a prime ideal of  $Z[i] / I$  ? what is the characteristic of  $Z[i] / I$  ?
- 8) show that  $Z_3[x] / \langle x^2 + x + 1 \rangle$  is not a field .
- 9) Prove that  $M$  is a maximal ideal in a ring  $R$  if and only if  $\forall x \notin M \exists r \in R$  such  $1 + r x \in M$  .